www.compassis.com

SeaFEM - Validation Case 9

Buoys joined with quasi-static elastic bars

SeaFEM

Version
15.1.0

http://www.compassis.com
info@compassis.com
November 2018

Table of Contents

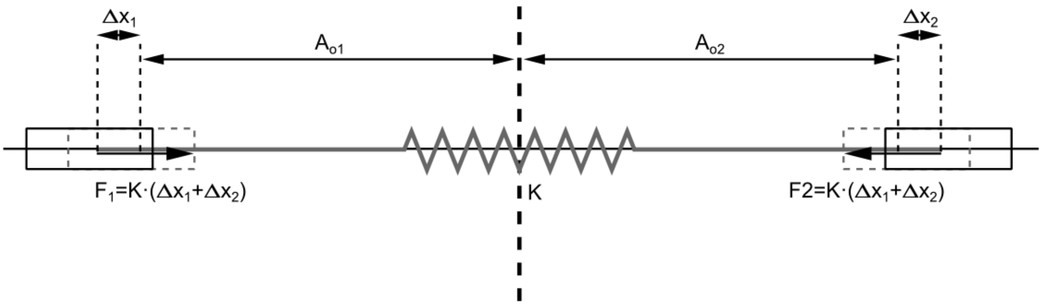
1 Validation Case 9 - Buoy quasi-static elastic bar

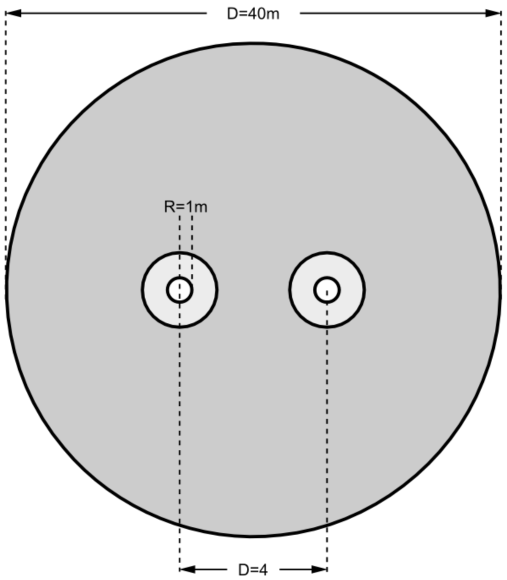
Harmonic oscillator

This validation case analyses the dynamic behavior of two buoys connected by an elastic bar. This simple model is aimed to verify some of the mooring capabilities of SeaFEM, in particular the elastic bar type of mooring segments.

For the sake of validation, we start from an equilibrium configuration consisting of two cylindrical bodies linked by an elastic bar attached at both ends to the gravity center of the floating bodies. This type of mooring or link segment acts effectively as an elastic spring able to work in both, tension and compression. For the sake of simplicity, the effect of waves is not taken into account. Hence, incident waves do not exist and the diffraction-radiation problem is not solved. Finally, the effective weight of the link segment is taken to be zero (neutrally buoyant). Under this conditions, the link segment does not introduce any vertical force component, and the floating bodies remain in hydrostatic equilibrium. Such a stable configuration is pertubed by initially applying a surge displacement to both floating bodies. Considering all these hypotheses, the model is reduced to a problem equivalent to a classical harmonic oscillator whose analytical solution [1] can be used for the sake of validation.

The following images show an sketch of the initial problem configuration and a top view of the computational domain respectively. The two cylinders have identical radius (R=1m) and draft (h=0.5 m) resulting on a displacement about 1.57 m^3 and a body mass about 1600 Kg (water density ρ = 1025 Kg/m^3).



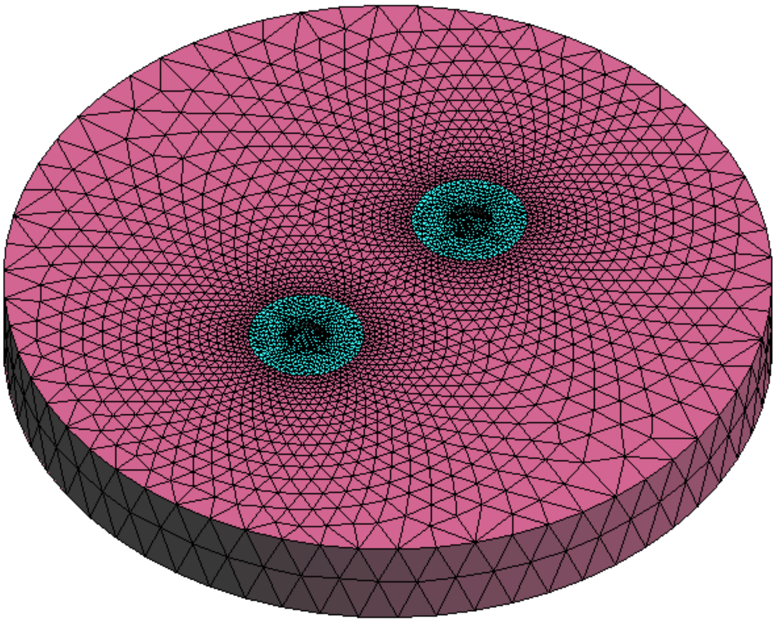


Mesh properties

Mesh properties for the present analysis are summarized in the following table:

Mesh properties	
Min. element size	0.2
Max element size	5
Mesh size transition	0.2
Number of elements	127355
Number of nodes	21845

Next picture shows an isometric view of the whole domain mesh used for the present analysis.



Solution to the harmonic oscillator problem.

The angular frequency of the system (ϖ) is given by:

$$\varpi = \left(\frac{2K}{M}\right)^{\frac{1}{2}}$$

where K is the elastic constant of the spring and $M = 1600$ Kg is the mass of each one of the cylinders.

The elastic constant K can be related to the mooring line segment properties as follows:

$$E = 3.0 \text{ GPa}$$

$$A = 3.75E\text{-}04\ m^2$$

$$L = 12\ m$$

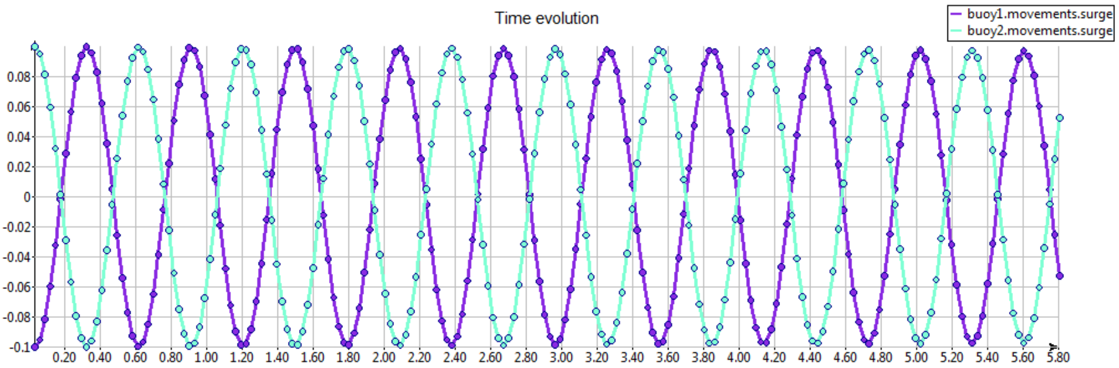
$$K = \frac{E \cdot A}{L} = 93750\ \frac{N}{m}$$

where E is the Young's modulus, A is the cross section of the elastic bar and L is the unstretched length.

If the two bodies are initially pulled apart and then let move free in the surge direction, they must oscillate in phase with an angular frequency $\varpi = (2K/M)^{(1/2)} = 10.7916$ rad/s

Results

The following picture shows the surge displacement response of the two floating bodies when they are initially pulled apart 0.1 meters from their equilibrium position:



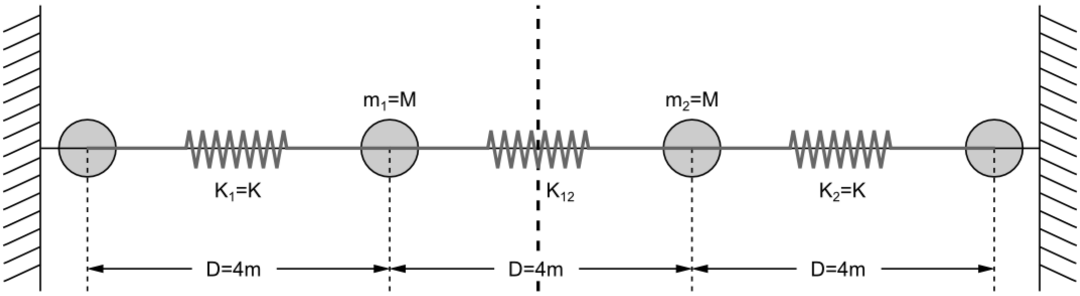
The angular frequency evaluated from these results is ω = 10.7025 rad/s. which matches the analytical value with an error less than one percent.

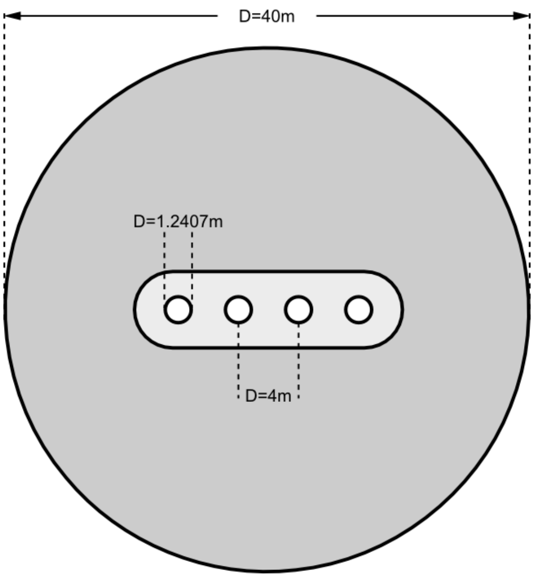
Coupled oscillators

A bit more interesting problem is analized next. In this case, 4 spherical bodies are aligned along the X-axis and linked together using the same type of elastic bars as in the previous test case. Two of the bodies, those located farther, remain fixed (all degrees of freedom are restricted), while the remaining ones are free to move along the surge direction. As well as in the previous case, all link segments are considered neutrally buoyant and the effect of waves is not taken into account.

Considering all these hypotheses, the model is equivalent to the classical problem of two coupled harmonic oscillators whose analytical solution [1] can be used for the sake of validation.

The following images show an sketch of the nitial problem configuration and a top view of the computational domain respectively. All spherical bodies have a radius R=0.62035 m with the center of mass located at the waterline level.



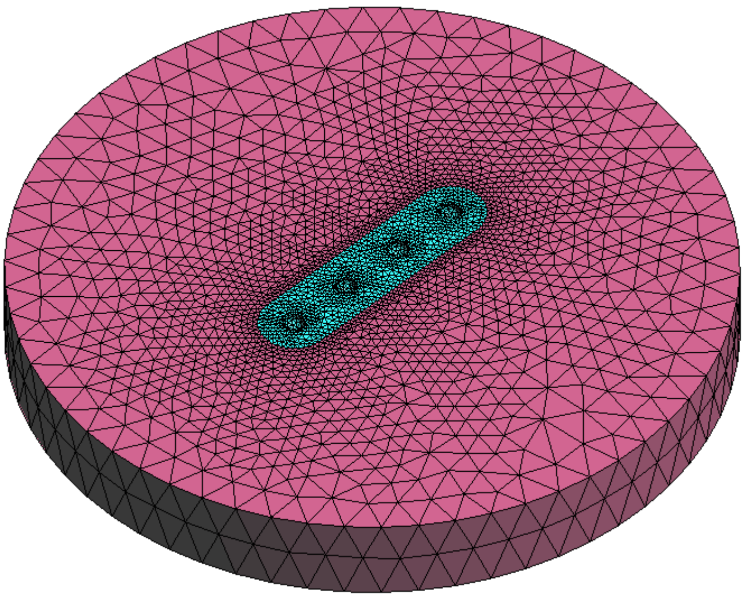


Mesh properties

Mesh properties for the present analysis are summarized in the following table:

Min. element size	0.2
Max element size	5
Mesh size transition	0.2
Number of elements	123881
Number of nodes	21239

Next picture shows an isometric view of the whole domain mesh used for the present analysis.



Solution to the coupled harmonic oscillator problem.

The model outline above corresponds to the simplest case of coupled oscillations, namely two identical harmonic oscillators connected by a spring, where $K_1 = K_2 = K$ is the recovery constant os each oscillator and K_{12} is the recovery constant of the coupling spring.

This kind of system has two characteristic frequencies:

$$\varpi_1 = \left(\frac{K + 2K_{12}}{M}\right)^{\frac{1}{2}}$$

$$\varpi_2 = \left(\frac{K}{M}\right)^{\frac{1}{2}}$$

Maybe the most interesting case occurs when the coupling between the two oscillators is weak, i.e. when $K_{12} << K$. In this case, the two characteristic frequencies become:

$$\varpi_1 = \varpi_0 \cdot (1 + \varepsilon)$$

$$\varpi_2 = \varpi_0 \cdot (1 - \varepsilon)$$

where

$$\varpi_0 = \left(\frac{K + K_{12}}{M}\right)^{\frac{1}{2}}$$

and $\varepsilon = (K_{12}/2K) << 1$

In the present simulation, the properties of the link segments where choosen as follows:

Cable segment cross section $A = 3.75E\text{-}4\ m^2$

Non-stretched cable length $L = 4.0\ m$

Effective weight $w = 0\ N/m$

Young's modulus $E_1 = E_2 = E = 3.0E9$ Pa and $E_{12} = 4.8E8$ Pa

This corresponds to the following values of the elastic recovery constants:

$$K_1 = K_2 = K = \frac{E \cdot A}{L} = 281250\ \frac{N}{m}$$

$$K_{12} = \frac{E_{12} \cdot A}{L} = 45000\ \frac{N}{m}$$

Hence, the characteristic frequencies of the movement are:

$$\varepsilon = 0.08$$

$$\varpi_1 \leq 27.249 \, \frac{\text{rad}}{s}$$

$$\varpi_2 \leq 23.717 \, \frac{\text{rad}}{s}$$

Now, we can analize the movement of the system. If we move the first oscillator a distance D and then let it go from rest, the initial conditions of the system are:

$$x_1(0) = D$$

$$x_2(0) = 0$$

$$v_1(0) = v_2(0) = 0$$

and the analytical solution becomes:

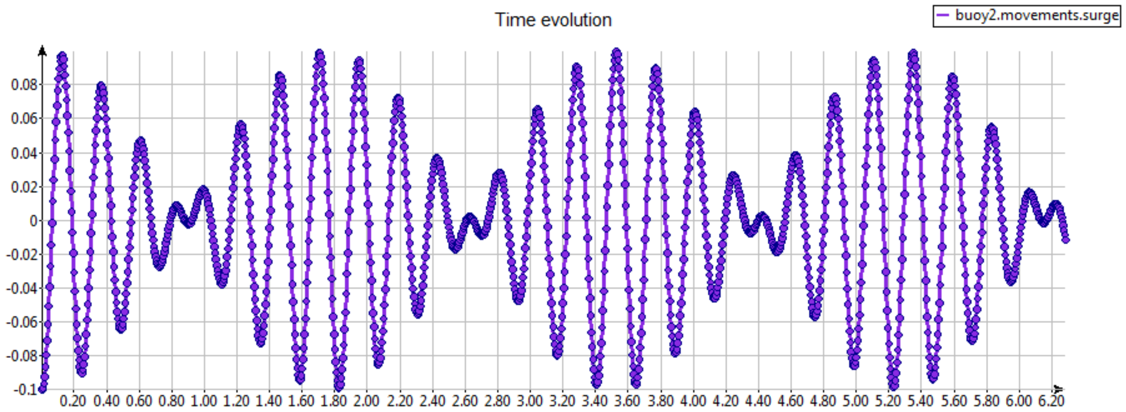
$$x_1(t) = D \cdot \cos(\varepsilon \cdot \varpi_0 \cdot t)) \cdot \cos(\varpi_0 \cdot t)$$

$$x_2(t) = D \cdot \sin(\varepsilon \cdot \varpi_0 \cdot t)) \cdot \sin(\varpi_0 \cdot t)$$

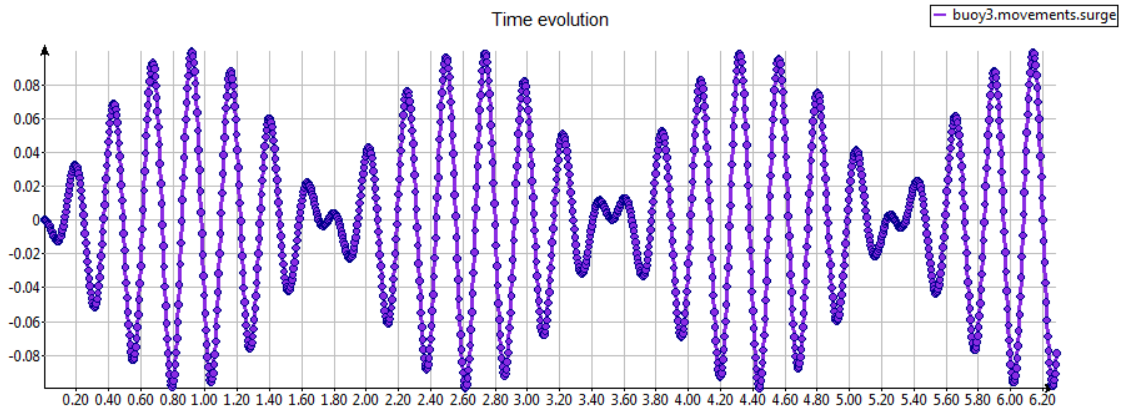
Since ε is small, the amplitudes of $x_1(t)$ and $x_2(t)$ vary slowly in time. At the initial time, only $x_1(t)$ is different from cero, but as time goes by the energy from one oscillator is transfered to the second one.

Results

The following picture shows the surge displacement response of the two oscillating spheres. It can be observed that the angular frequency of both oscillators closely matches that predicted by the analytical solution. The time variation of the oscillators amplitude also follows the expected behavior.



Surge displacement of oscillator number 1 (initial displacement = -0.1 m)



Surge displacement of oscillator number 2 (initial displacement = 0.0 m)

Appendix

This appendix is devoted to present the tcl code necessary to define the mooring segments used to link various bodies in both cases analized in the present test case. Note that in newer versions of the program mooring lines can be already defined using the graphic user interface of SeaFEM. In this case, the `TdynTcl_CreateMooring` procedure must be commented in the Tcl script to avoid mooring duplication. On the contrary, `TdynTcl_InitiateProblem` procedure must remain active within the Tcl script to configure the solver so that the diffraction-radiation problem is not effectively solved.

Harmonic oscillator case

```
proc TdynTcl_InitiateProblem { } {
   configure_analysis Solve_Dif_Rad 0
}

proc TdynTcl_CreateMooring { } {
   set area 3.75E-4
   set L 12
   set w 0.0

   set seg1 [TdynTcl_Create_Mooring_Segment 1 -6.0 0.0 0.0 6.0 0.0 0.0 $w $L $area
3.0E+9 0]

   TdynTcl_Create_Mooring_Link $seg1 2

   TdynTcl_Message "TdynTcl_CreateMooringLine finished!!!" notice
}
```

Coupled oscillators case

```
proc TdynTcl_InitiateProblem { } {
   configure_analysis Solve_Dif_Rad 0
}

proc TdynTcl_CreateMooring { } {
   set area 3.75E-4
   set L 4.0
```

```
    set w 0

    set seg1   [TdynTcl_Create_Mooring_Segment 1 1 -6.0 0.0 0.0 -2.0 0.0 0.0 $w $L $area
3.0E+9 0]
    set seg2   [TdynTcl_Create_Mooring_Segment 2 1 -2.0 0.0 0.0 2.0 0.0 0.0 $w $L $area
4.8E+8 0]
    set seg3   [TdynTcl_Create_Mooring_Segment 3 1 2.0 0.0 0.0 6.0 0.0 0.0 $w $L $area
3.0E+9 0]

    TdynTcl_Create_Mooring_Link $seg1 2
    TdynTcl_Create_Mooring_Link $seg2 3
    TdynTcl_Create_Mooring_Link $seg3 4

    TdynTcl_Message "TdynTcl_CreateMooringLine finished!!!" notice
}
```

References

[1] S. T. Thornton and J. B. Marion, Classical dynamics of particles and systems.

Validation Summary

CompassFEM version	15.1.0
Tdyn solver version	15.1.0
RamSeries solver version	15.1.0
Benchmark status	Successfull
Last validation date	27/11/2018