RamSeries - Validation Case 47

Laminated shell stiffness matrix characteristics

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1. Validation Case 47

1.1. Model Description

Basic validation and verification of the laminate's analysis models implemented in a FEA software must consider the assessment of the membrane and bending stiffness characteristics and the membrane and bending strength analysis [1]. The present document addresses the first of these issues (i.e. the membrane and bending stiffness characteristics) to assess the behavior of the classical lamination theory (CLT) implemented in RamSeries. To this aim, the benchmark problems provided in [1] are reproduced. The stiffness matrix characteristics obtained with RamSeries are compared againsts the results reported in [1].

The examples presented here comprise laminates that exhibit no coupling in the stiffness matrix with both, isotropic and orthotropic plies. A number of tests with stacking sequences leading to different forms of coupling are also presented.

In all cases, a single 4-noded quadrilateral element is used for shell discretization. An schematic representation of the shell geometry under analysis is presented in Fig.1.

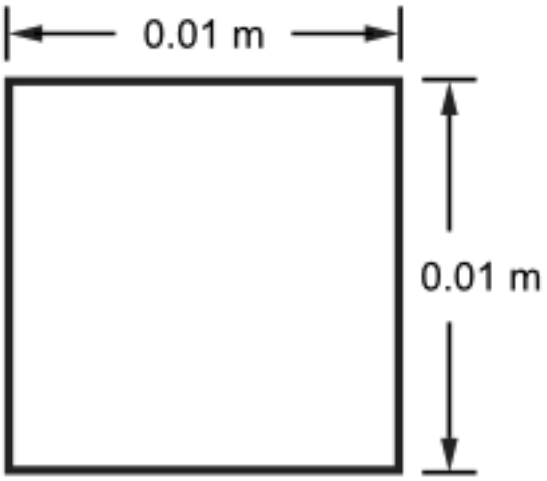

Fig.1. Dimensions of the laminated shell under analysis. The shell is discretized using a single quadrilateral element.

Laminate definition and material properties are summarized for each test case in the following tables.

Benchmark name Material data	LSMC1	
El (N/m^2)	7.20E+10	
Et (N/m^2)	7.20E+10	
ν_{tl}	0.3	
G_{lt} N/m^2	2.76E+10	
G_{lz} N/m^2	2.76E+10	
G_{tz} N/m^2	2.76E+10	
α_l	2.25E+07	
α_t	2.25E+07	
σ_{lt} N/m^2	5.00E+08	
σ_{lc} N/m^2	3.56E+09	
σ_{tt} N/m^2	5.00E+06	
σ_{tc} N/m^2	7.50E+07	
σ_{shr} N/m^2	3.50E+07	
Laminate definition ply number	ply t (m)	ply theta
1	1.00E-04	0
2	1.00E-04	0
3	1.00E-04	0
4	1.00E-04	0

Table 1. Material properties and laminate's definition for case 1. It corresponds to a simple laminate consisting in four plies of isotropic material with all plies oriented at zero degrees with respect to the laminate's axes.

Benchmark name Material data	LSMC2	
El (N/m^2)	2.13E+11	
Et (N/m^2)	8.20E+09	
ν_{tl}	0.3	
G_{lt} N/m^2	3.20E+09	
G_{lz} N/m^2	3.20E+09	
G_{tz} N/m^2	3.20E+09	
α_l	1.30E-06	
α_t	2.70E-05	
σ_{lt} N/m^2	5.00E+08	
σ_{lc} N/m^2	3.50E+08	
σ_{tt} N/m^2	5.00E+06	
σ_{tc} N/m^2	7.50E+07	
σ_{shr} N/m^2	3.50E+07	
Laminate definition ply number	ply t (m)	ply theta
1	1.00E-04	0
2	1.00E-04	0
3	1.00E-04	0
4	1.00E-04	0

Table 2. Material properties and laminate's definition for case 2. It corresponds to a simple laminate consisting in four plies of orthotropic material with all plies oriented at zero degrees with respect to the laminate's axes.

Benchmark name Material data	LSMC3	
El (N/m^2)	2.13E+11	
Et (N/m^2)	8.20E+09	
ν_{tl}	0.3	
G_{lt} N/m^2	3.20E+09	
G_{lz} N/m^2	3.20E+09	
G_{tz} N/m^2	3.20E+09	
α_l	1.30E-06	
α_t	2.70E-05	
σ_{lt} N/m^2	5.00E+08	
σ_{lc} N/m^2	3.50E+08	
σ_{tt} N/m^2	5.00E+06	
σ_{tc} N/m^2	7.50E+07	
σ_{shr} N/m^2	3.50E+07	
Laminate definition ply number	ply t (m)	ply theta
1	1.00E-04	0
2	1.00E-04	60
3	1.00E-04	-60
4	1.00E-04	0
5	1.00E-04	0
6	1.00E-04	-60
7	1.00E-04	60
8	1.00E-04	0

Table 3. Material properties and laminate's definition for case 3. It corresponds to a symmetric balanced laminate with plies of orthotropic material.

Benchmark name Material data	LSMC4	
El (N/m^2)	7.80E+06	
Et (N/m^2)	2.60E+06	
ν_{tl}	0.25	
G_{lt} N/m^2	1.25E+06	
G_{lz} N/m^2	1.25E+06	
G_{tz} N/m^2	1.25E+06	
α_l	3.50E-06	
α_t	1.14E-05	
σ_{lt} N/m^2	5.00E+08	
σ_{lc} N/m^2	3.50E+08	
σ_{tt} N/m^2	5.00E+06	
σ_{tc} N/m^2	7.50E+07	
σ_{shr} N/m^2	3.50E+07	
Laminate definition ply number	ply t (m)	ply theta
1	0.09	0
2	0.09	90

Table 4. Material properties and laminate's definition for case 4. It corresponds to a cross-ply laminate with plies of orthrtropic material. It results in a laminate with membrane/bending coupling.

Benchmark name Material data	LSMC5	
El (N/m^2)	2.13E+11	
Et (N/m^2)	8.20E+09	
ν_{tl}	0.3	
G_{lt} N/m^2	3.20E+09	
G_{lz} N/m^2	3.20E+09	
G_{tz} N/m^2	3.20E+09	
α_l	1.30E-06	
α_t	2.70E-05	
σ_{lt} N/m^2	5.00E+08	
σ_{lc} N/m^2	3.50E+08	
σ_{tt} N/m^2	5.00E+06	
σ_{tc} N/m^2	7.50E+07	
σ_{shr} N/m^2	3.50E+07	
Laminate definition ply number	ply t (m)	ply theta
1	5.00E-05	45
2	5.00E-05	90
3	5.00E-05	60

Table 5. Material properties and laminate's definition for case 5. It corresponds to an unsymmetric unbalanced laminate with plies of orthrtropic material. It results in a fully populated membrane/bending coupling matrix.

Benchmark name Material data	LSMC6	
El (N/m^2)	2.07E+11	
Et (N/m^2)	7.60E+09	
ν_{tl}	0.3	
G_{lt} N/m^2	5.00E+09	
G_{lz} N/m^2	5.00E+09	
G_{tz} N/m^2	5.00E+09	
α_l	0	
α_t	3.00E-05	
σ_{lt} N/m^2	5.00E+08	
σ_{lc} N/m^2	3.50E+08	
σ_{tt} N/m^2	5.00E+06	
σ_{tc} N/m^2	7.50E+07	
σ_{shr} N/m^2	3.50E+07	
Laminate definition ply number	ply t (m)	ply theta
1	5.00E-05	90
2	5.00E-05	-45
3	5.00E-05	45
4	5.00E-05	0

Table 6. Material properties and laminate's definition for case 6. It corresponds to an unsymmetric laminate with plies of orthrtropic material, exhibiting membrane-bending coupling behavior.

1.2. Results

For the sake of validation, the terms of the membrane, bending and coupling modulus matrix obtained with RamSeries for each laminate are compared with the corresponding values reported in the NAFEMS document [1].

Benchmark name	LSMC1				
Ply angles	[0/0/0/0]				
	Theory	MSC/NASTRAN 2001	Error %	RamSeries	Error %
Membrane modulus matrix					
A11	7.9120E+10	7.9121E+10	0.0013	7.9121E+10	0.0011
A12	2.3740E+10	2.3760E+10	0.0842	2.3736E+10	0.0158
A13	0.0000E+00			0.0000E+00	
A22	7.9121E+10	7.9121E+10	0.0000	7.9121E+10	0.0002
A23	0.0000E+00			0.0000E+00	
A33	2.7600E+10	2.7690E+10	0.3261	2.7600E+10	0.0000
Bending modulus matrix					
D11	7.9120E+10	7.9121E+10	0.0013	7.9121E+10	0.0011
D12	2.3740E+10	2.3760E+10	0.0842	2.3736E+10	0.0158
D13	0.0000E+00			0.0000E+00	
D22	7.9121E+10	7.9121E+10	0.0000	7.9121E+10	0.0002
D23	0.0000E+00			0.0000E+00	
D33	2.7600E+10	2.7690E+10	0.3261	2.7600E+10	0.0000
Coupling modulus matrix					
B11	-	-	-	-1.4211E-06	-
B12	-	-	-	0.0000E+00	-
B13	-	-	-	0.0000E+00	-
B22	-	-	-	-1.4211E-06	-
B23	-	-	-	0.0000E+00	-
B33	-	-		0.0000E+00	-

Benchmark name	LSMC2				
Ply angles	[0/0/0/0]				
	Theory	MSC/NASTRAN 2001	Error %	RamSeries	Error %
Membrane modulus					
A11	2.1370E+11	2.1374E+11	0.0187	2.1374E+11	0.0190
A12	2.4690E+09	2.4686E+09	0.0162	2.4686E+09	0.0181
A13	0.0000E+00			0.0000E+00	
A22	8.2290E+09	8.2285E+09	0.0061	8.2285E+09	0.0060
A23	0.0000E+00			0.0000E+00	
A33	3.2000E+09	3.2000E+09	0.0000	3.2000E+09	0.0000
Bending modulus m					
D11	2.1370E+11	2.1374E+11	0.0187	2.1374E+11	0.0190
D12	2.4690E+09	2.4686E+09	0.0162	2.4686E+09	0.0181
D13	0.0000E+00			0.0000E+00	
D22	8.2290E+09	8.2285E+09	0.0061	8.2285E+09	0.0060
D23	0.0000E+00			0.0000E+00	
D33	3.2000E+09	3.2000E+09	0.0000	3.2000E+09	0.0000
Coupling modulus m					
B11	-	-	-	-	-
B12	-	-	-	-	-
B13	-	-	-	-	-
B22	-	-	-	-	-
B23	-	-	-	-	-
B33	-	-		-	-

Benchmark name	LSMC3				
Ply angles	[0/60/-60/0/0/-60/60/0]				
	Theory	MSC/NASTRAN 2001	Error %	RamSeries	Error %
Membrane modulus					
A11	1.1750E+11	1.1753E+11	0.0255	1.1753E+11	0.0228
A12	2.1620E+10	2.1615E+10	0.0231	2.1615E+10	0.0217
A13	0.0000E+00			0.0000E+00	
A22	6.6150E+10	6.6149E+10	0.0015	6.6149E+10	0.0018
A23	0.0000E+00			0.0000E+00	
A33	2.2350E+10	2.2347E+10	0.0134	2.2347E+10	0.0145
Bending modulus m					
D11	1.3560E+11	1.3557E+11	0.0221	1.3557E+11	0.0244
D12	1.8030E+10	1.8025E+10	0.0277	1.8025E+10	0.0261
D13	4.1970E+09	4.1974E+09	0.0095	4.1974E+09	0.0085
D22	5.5290E+10	5.5289E+10	0.0018	5.5289E+10	0.0023
D23	1.2490E+10	1.2488E+10	0.0160	1.2488E+10	0.0149
D33	1.8760E+10	1.8757E+10	0.0160	1.8757E+10	0.0174
Coupling modulus m					
B11	-	-	-	4.2633E-06	-
B12	-	-	-	-4.8850E-07	-
B13	-	-	-	0.0000E+00	-
B22	-	-	-	-2.7534E-06	-
B23	-	-	-	3.5527E-07	-
B33	-	-		-2.4425E-07	-

Benchmark name	LSMC4				
Ply angles	[0/90]				
	Theory	MSC/NASTRAN 2001	Error %	RamSeries	Error %
Membrane modulus matrix					
A11	5.3110E+06	5.3106E+06	0.0075	5.3106E+06	0.0068
A12	6.6380E+05	6.6384E+05	0.0060	6.6383E+05	0.0045
A13	0.0000E+00	1.5568E-11	-	0.0000E+00	-
A22	5.3110E+06	5.3106E+06	0.0075	5.3106E+06	0.0068
A23	0.0000E+00	1.4702E-10	-	0.0000E+00	-
A33	1.2500E+06	1.2500E+06	0.0000	1.2500E+06	0.0000
Bending modulus matrix					
D11	5.3110E+06	5.3106E+06	0.0075	5.3106E+06	0.0068
D12	6.6380E+05	6.6383E+05	0.0045	6.6383E+05	0.0045
D13	0.0000E+00	1.5568E-11	-	0.0000E+00	
D22	5.3110E+06	5.3106E+06	0.0075	5.3106E+06	0.0068
D23	0.0000E+00	1.4702E-10	-	0.0000E+00	
D33	1.2500E+06	1.2500E+06	0.0000	1.2500E+06	0.0000
Coupling modulus matrix					
B11	6.6383E+05	6.6383E+05	0.0000	6.6383E+05	0.0000
B12	0.0000E+00	-	-	0.0000E+00	-
B13	0.0000E+00	-3.8920E-12	-	0.0000E+00	-
B22	-6.6383E+05	-6.6383E+05	0.0000	-6.6383E+05	0.0000
B23	0.0000E+00	-3.6754E-11	-	0.0000E+00	-
B33	0.0000E+00	-	-	0.0000E+00	-

Benchmark name	LSMC5				
Ply angles	[45/90/60]				
	Theory	MSC/NASTRAN 2001	Error %	RamSeries	Error %
Membrane modulus					
A11	2.9820E+10	2.9823E+10	0.0101	2.9823E+10	0.0091
A12	3.2250E+10	3.2252E+10	0.0062	3.2252E+10	0.0074
A13	2.4590E+10	2.4588E+10	0.0081	2.4588E+10	0.0083
A22	1.3260E+11	1.3258E+11	0.0151	1.3258E+11	0.0161
A23	3.9330E+10	3.9327E+10	0.0076	3.9327E+10	0.0072
A33	3.2980E+10	3.2984E+10	0.0121	3.2984E+10	0.0116
Bending modulus m					
D11	3.9420E+10	3.9420E+10	0.0000	3.9420E+10	0.0003
D12	4.5490E+10	4.5490E+10	0.0000	4.5490E+10	0.0008
D13	3.5520E+10	3.5516E+10	0.0113	3.5516E+10	0.0114
D22	9.6510E+10	9.6507E+10	0.0031	9.6507E+10	0.0033
D23	5.6810E+10	5.6806E+10	0.0070	5.6806E+10	0.0073
D33	4.6220E+10	4.6221E+10	0.0022	4.6221E+10	0.0024
Coupling modulus m					
B11	4.2900E+09	4.2904E+09	0.0093	4.2904E+09	0.0091
B12	1.4183E+09	1.4183E+09	0.0000	1.4183E+09	0.0016
B13	3.2213E+09	3.2213E+09	0.0000	3.2213E+09	0.0014
B22	-7.1269E+09	-7.1270E+09	0.0014	-7.1269E+09	0.0006
B23	-1.6917E+09	-1.6917E+09	0.0000	-1.6917E+09	0.0007
B33	1.4183E+09	1.4183E+09	0.0000	1.4183E+09	0.0016

Benchmark name	LSMC6				
Ply angles	[90/-45/45/0]				
	Theory	MSC/NASTRAN 2001	Error %	RamSeries	Error %
Membrane modulus					
A11	8.3810E+10	8.3814E+10	0.0048	8.3814E+10	0.0044
A12	2.6130E+10	2.6130E+10	0.0000	2.6130E+10	0.0015
A13	0.0000E+00	-	-	0.0000E+00	-
A22	8.3810E+10	8.3814E+10	0.0048	8.3814E+10	0.0044
A23	0.0000E+00	2.3283E-06	-	2.3283E-06	-
A33	2.8840E+10	2.8842E+10	0.0069	2.8842E+10	0.0071
Bending modulus m					
D11	1.0170E+11	1.0170E+11	0.0000	1.0170E+11	0.0047
D12	8.2480E+09	8.2481E+09	0.0012	8.2481E+09	0.0008
D13	0.0000E+00			2.6021E-06	
D22	1.0170E+11	1.0170E+11	0.0000	1.0170E+11	0.0047
D23	0.0000E+00	5.2042E-06		1.9516E-06	
D33	1.0960E+10	1.0961E+10	0.0091	1.0961E+10	0.0047
Coupling modulus m					
B11	-1.8756E+10	-1.8756E+10	0.0000	-1.8756E+10	0.0014
B12	-3.8520E-26	1.3323E-07	-	-5.3291E-07	-
B13	-3.1260E+09	-3.1260E+09	0.0000	-3.1260E+09	0.0015
B22	1.8756E+10	1.8756E+10	0.0000	1.8756E+10	0.0014
B23	-3.1260E+09	-3.1260E+09	0.0000	-3.1260E+09	0.0015
B33	0.0000E+00	8.8818E-08	-	-7.1054E-07	-

1.3. Validation Summary

CompassFEM version	**15.1.0**
Tdyn solver version	15.1.0
RamSeries solver version	15.1.0
Benchmark status	Successfull
Last validation date	27/11/2018

1.4. References

[1] P. Hopkins, “Benchmarks for Membrane and bending analysis of Laminated Shells. Part 1. Stiffness matrix and thermal characteristics.,” NAFEMS, 2005.