1 Flow through porous media

Introduction

This report shows the capabilities of Tdyn for modelling the fluid flow through porous media.

Modelling approach

Modeling of flows through a porous medium requires a modified formulation of the Navier-Stokes equations, which reduces to their classical form and includes additional resistance terms induced by the porous region.

The incompressible Navier-Stokes equations in a given domain Ω and time interval (0,t) can be written as:

$$\rho\left(\frac{\partial u}{\partial t} + (u \cdot \nabla) u\right) - \mu \nabla^2 u + \nabla p = f \text{ on } \Omega \text{x}(0,t)$$
$$\nabla \cdot u = 0 \text{ on } \Omega \text{x}(0,t)$$

where $u = u(x,t)$ denotes the velocity vector, $p = p(x,t)$ the pressure field, ρ the constant density, μ the dynamic viscosity coefficient and f represents the external body forces acting on the fluid (i.e. gravity).

In the case of solids, small velocities can be considered and $(u \cdot \nabla)u$ term is neglected. Therefore, assuming an incompressible flow (constant density) in a certain domain Ω and considering the mass conservation equation, also called continuity equation, the Navier-Stokes equation can be written as follows:

$$\rho \frac{\partial u}{\partial t} - \mu \nabla^2 u + \nabla p = f \text{ on } \Omega \text{x}(0,t)$$
$$\nabla \cdot u = 0 \text{ on } \Omega \text{x}(0,t)$$

The general form of the Navier-Stokes equation is valid for the flow inside pores of the porous medium but its solution cannot be generalized to describe the flow in porous region. Therefore, the general form of Navier-Stokes equation must be modified to describe the flow through porous media. To this aim, Darcy's law is used to describe the linear relation between the velocity u and gradient of pressure p in the porous medium. It defines the permeability resistance of the flow in a porous media:

$$\nabla p = -\mu D u \text{ in } \Omega_p x(0,t)$$

where Ω_p is the porous domain, D the Darcy's law resistance matrix and u the velocity vector. In the case of considering an homogeneous porous media, D is a diagonal matrix with coefficients $1/\alpha$, where α is the permeability coefficient.

The Reynolds number is defined as:

$$Re = \frac{\rho U L}{\mu}$$

being U and L a characteristic velocity and a characteristic length scale, respectively. In order to characterize the inertial effects, it is possible to define the Reynolds number associated to the pores:

$$Re_p = \frac{\rho U \delta}{\mu}$$

where δ is the characteristic pore size. Whereas Darcy law is reliable for values of $Re_p < 1$, otherwise it is necessary to consider a more general model which accounts also for the inertial effects, such as:

$$\nabla p = -\left(\mu D u + \frac{1}{2}\rho C u|u|\right) \text{ in } \Omega_p x(0,t)$$

being C the inertial resistance matrix.

Considering a modified Navier-Stokes equation in the whole domain including the two source terms associated to the resistance induced by the porous medium (linear Darcy and inertial loss term), the momentum equations become:

$$\rho\frac{\partial u}{\partial t} - \mu\nabla^2 u + \nabla p - \mu D u - \frac{1}{2}\rho C u|u| = 0 \text{ in } \Omega x(0,t)$$

In the case of considering an homogeneous porous media, D is a diagonal matrix with coefficients $1/\alpha$, where α is the permeability coefficient and C is also a diagonal matrix. Therefore, a modified Darcy's resistance matrix should be used in Tdyn, as follows:

$$\rho\frac{\partial u}{\partial t} - \mu\nabla^2 u + \nabla p = \mu D^* u \text{ in } \Omega x(0,t)$$

$$D^* = D + \frac{1}{2}\mu\,\rho|u|I$$

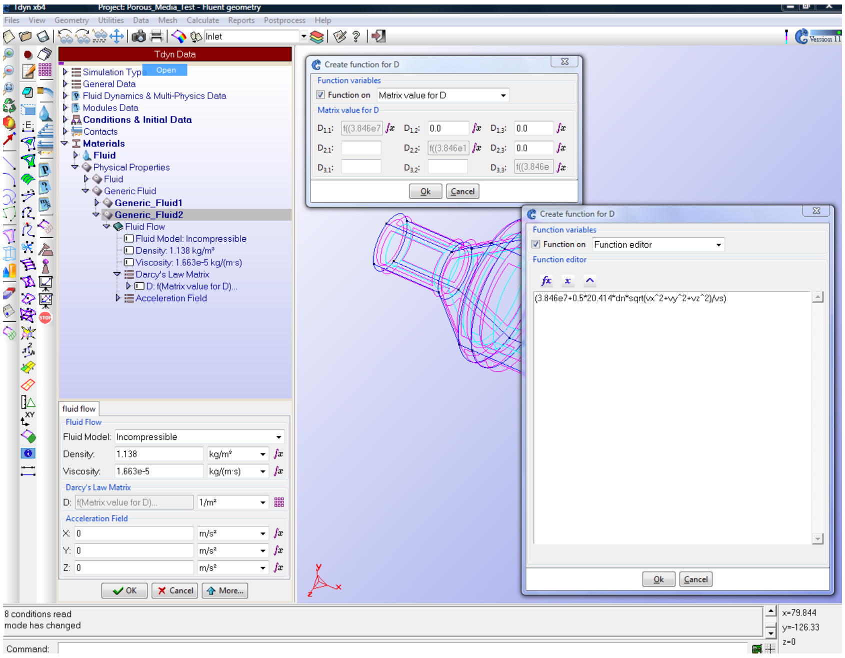
where I is the identity matrix. It should be noted that in laminar flows through porous media, the pressure p is proportional to velocity u and C can be considered zero ($D^* = D$). Therefore, the Navier-Stokes momentum equations can be rewritten as:

$$\rho\frac{\partial u}{\partial t} - \mu\nabla^2 u + \nabla p = -\mu D u \text{ in } \Omega x(0,t)$$

Remark1: These momentum equations are resolved by Tdyn in the case of solids, where $(u\cdot\nabla)u$ term vanishes due to small velocities. If $(u\cdot\nabla)u = 0$ can't be neglected in the modelization (i.e. high velocities), then Tdyn should resolve the following momentum equations in a fluid instead of in a solid:

$$\rho\left(\frac{\partial u}{\partial t} + (u\,.\,\nabla)u\right) - \mu\nabla^2 u + \nabla p = -\mu D u \text{ in } \Omega x(0,t)$$

Remark 2: In order to add the Darcy's resistance matrix D^* an appropriate function should be inserted in Tdyn. See the figure below and "Function syntax" in the Help manual of Tdyn for more details about defining functions.

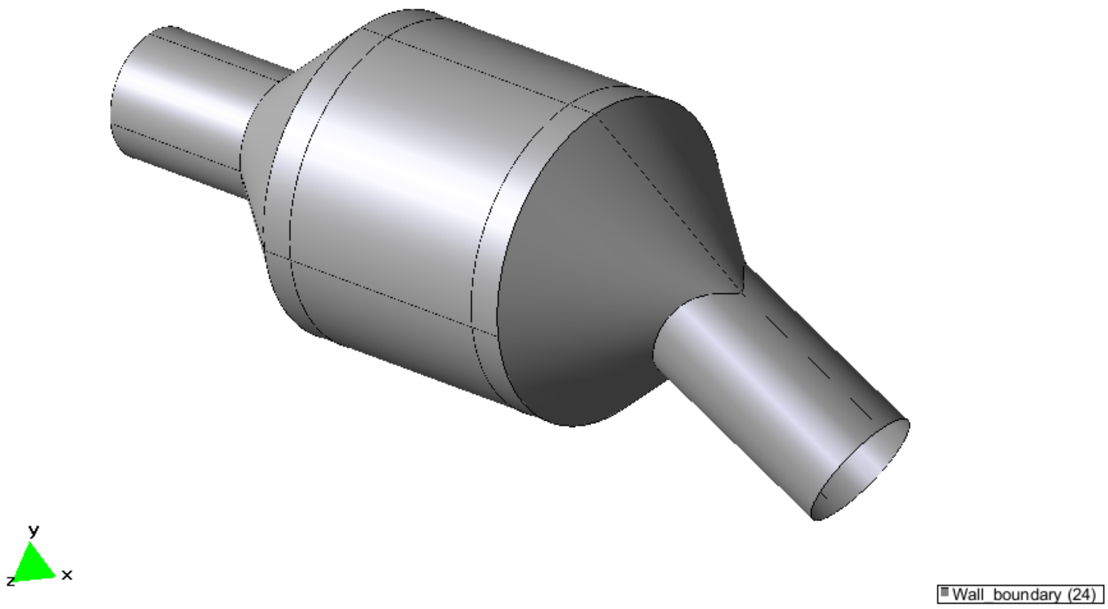


2 Test case

The following test case concerns the analysis of a gas flowing through a catalytic converter. The main objective is to determine the pressure gradient and the velocity distribution through the porous media that fills the catalytic element. This model is intended to exemplify the use of Tdyn capabilities to solve problems involving flow through porous media.

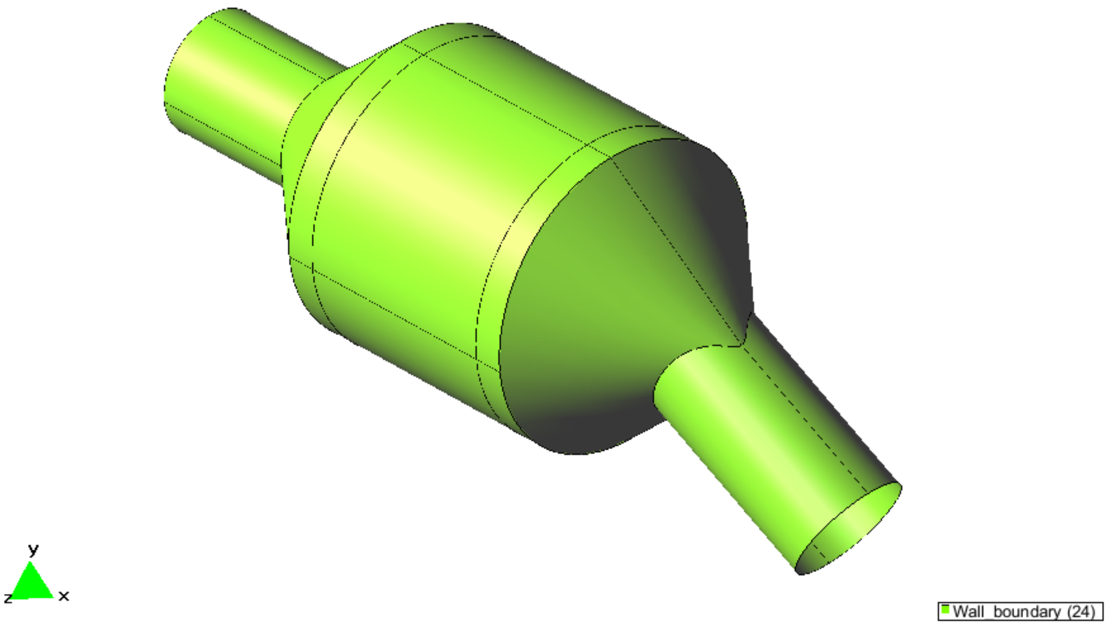
Geometry and boundary conditions

The following picture shows the geometry of the catalytic converter. The corresponding GiD model can be downloaded from the following link at the Compass web page http://www.compassis.com/downloads/Manuals/Porous_media.gid.zip.

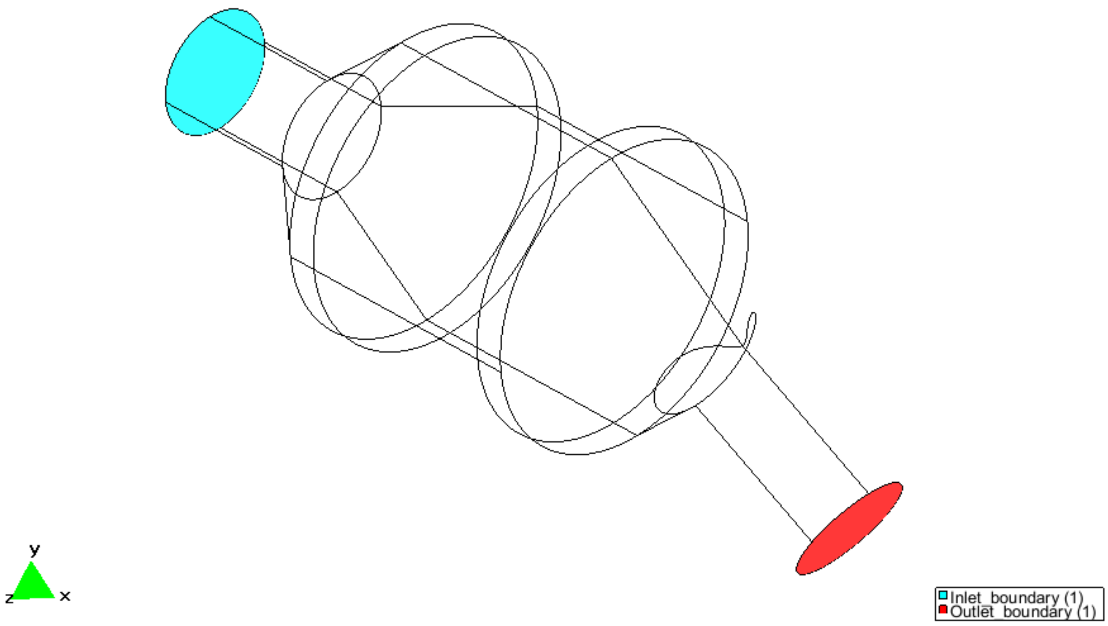


Test case

The boundary conditions are applied as shown in the figures below. First, a **Wall/Body** condition with a **Y-Plus wall** boundary type is applied to the external surface of the catalytic converter (in green in the first picture below).



A **Fix Velocity** condition is further applied to the inlet surface of the model (in blue in the figure below) so that the velocity vector is fixed there to the value **v = (22.6, 0.0, 0.0) m/s**. Finally, a **Pressure Field** condition is used to fix the the dynamic pressure to zero at the outlet surface of the control volume (in red in the figure below).



Materials

Two different materials are used in order to simulate the gas flowing through the catalyst converter. Fluid flow properties, density and viscosity, are the same in both materials and correspond to those of nitrogen

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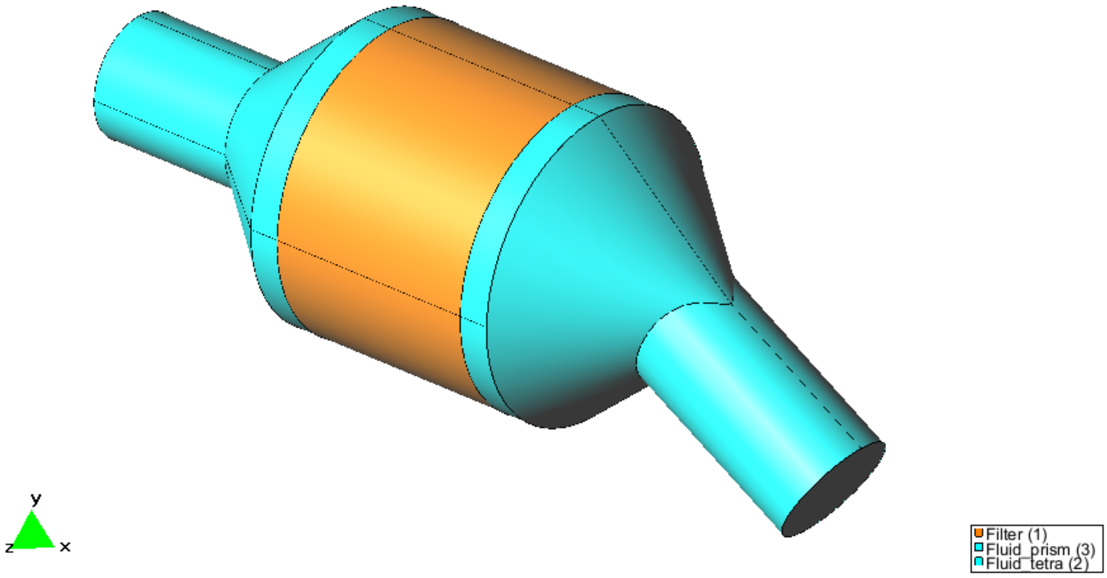
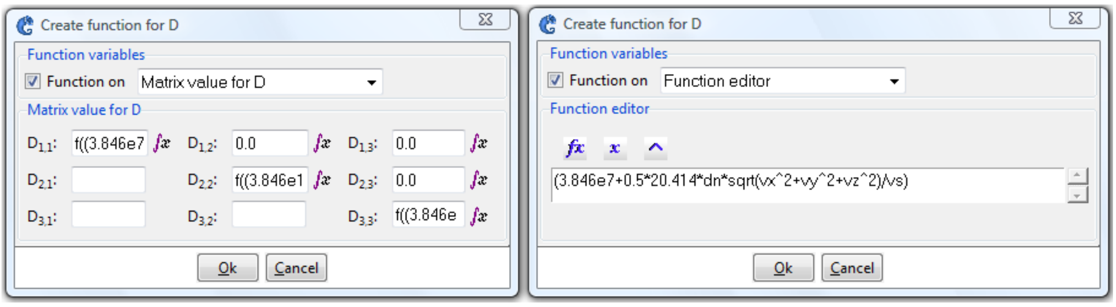
gas. The two materials only differ on the values of the Darcy's law matrix. While the first one has a null matrix, the second one has assigned a Darcy matrix incorporating the resistance and inertial effects of the porous media. Hence, the first material is applied to the inlet and outlet channel regions of the catalyst converter (in blue in the figure below), while the second one is applied to the intermidiate volume corresponding to the porous media (in yellow in the figure below).



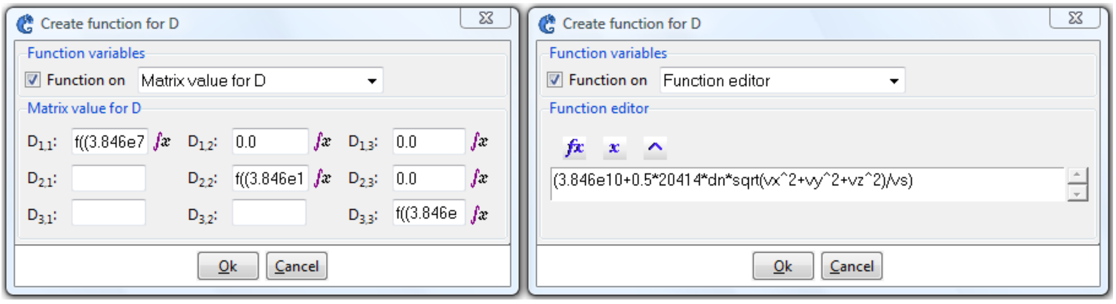
The Darcy's law matrix used to model the porous media is taken to be diagonal but anisotropic. The corresponding values of the viscous (D) and inertial (C) resistance terms are those in the table below. Note that the components of the Darcy's matrix are given with respect to the global axis system (X, Y, Z) so that the geometry must be conviniently oriented.

	D (1/m^2)	**C (1/m)**
X	3.846 x 10^7	20.414
Y	3.846 x 10^{10}	20414
Z	3.846 x 10^{10}	20414

With those values, the entries of the modified Darcy's matrix must be introduced as shown in the following pictures. The first function corresponds to $D_{1,1}$ component, while the second function corresponds to $D_{2,2}$ and $D_{3,3}$ components.



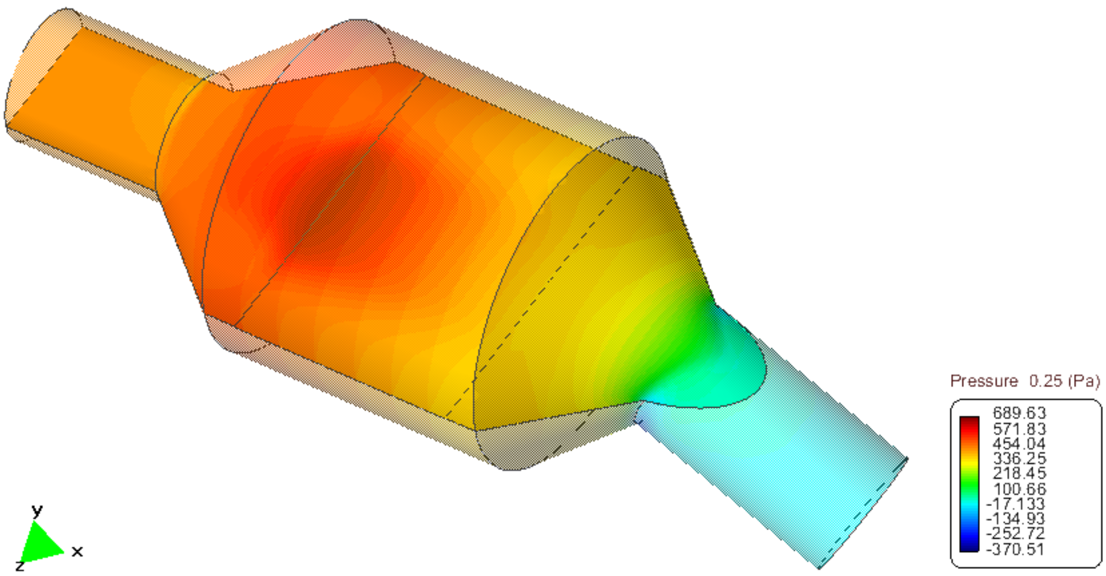
Test case



Results

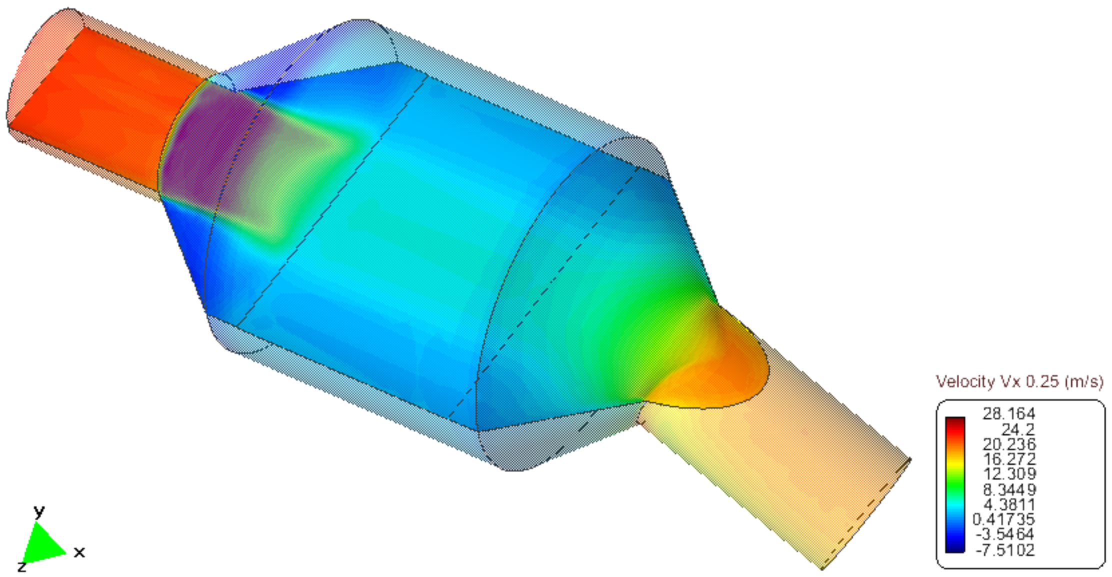
In this section, the results of the simulation are presented.

The pressure field distribution is shown in the first picture below. There, it can be observed that an important pressure drop occurs within the porous region.



The second picture shows the X-velocity component distribution of the fluid flow. In this picture, it becomes evident how the fluid decelerates rapidly when entering to the porous region. It can also be observed how the fluid recirculates before entering the central region of the catalyst (negative values of X-velocity component in dark blue) due to the resistance exherted by the porous media.

Test case



3 Model

Porous_media.gid.zip

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